Coulomb blockade and Quantum Critical Points in Quantum Dots

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Abstract

An ultrasmall quantum dot coupled to a lead and to a quantum box (a large quantum dot) is investigated. Tuning the tunneling amplitudes to the lead and box, we find a line of unstable non-Fermi-liquid fixed points as function of the gate potentials of the quantum dots, extending to arbitrary charging energies on the small and large quantum dots. These quantum-critical fixed points possess a finite residual entropy. They govern the cross over from one Fermi-liquid regime to another, characterized by distinct (high and low) conductance values.

Key words: Quantum dot, Two-channel Kondo effect, Numerical Renormalization Group

Strongly correlated electron systems display usual non-Fermi-liquid behavior in the vicinity of zero-temperature phase transitions [1,2]. The nature of these so-called quantum critical points is still not well understood in real materials. The two-channel Kondo effect (2CKE) is a prototype for such a quantum critical point in a quantum impurity system. It occurs when a spin-$\frac{1}{2}$ local moment is coupled antiferromagnetically with equal strength to two independent conduction-electron channels that overscreen its moment [3]. Its fixed point governs the transition between two distinct Fermi liquids, which are adiabatically connected at finite temperature.

The remarkable capabilities of detailed sample engineering and the direct control of the microscopic model parameters have turned quantum-dot devices into an important tool for investigating fundamental questions such as the 2CKE. We explore a double-dot device, comprised of a quantum box (i.e., a large dot) indirectly coupled to a lead via an ultrasmall quantum dot [4], see Fig.1. The Coulomb blockade on the quantum box suppresses charge fluctuations at temperature below the charging energy E_C of the box, dynamically generating two independent screening channels for the spin on the ultrasmall quantum dot [5]. The latter dot is modeled by a single energy level ϵ_d and an on-site Coulomb energy U, embedded between a metallic lead and a quantum box. The quantum box is characterized by a dense set of single-particle levels. Denoting the creation of an electron with spin projection σ on the dot by $d_\sigma^\dagger$, the corresponding Hamiltonian reads

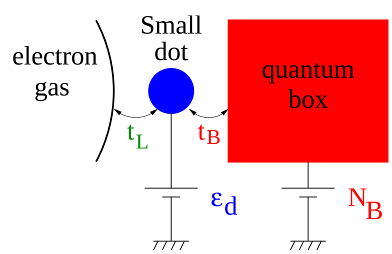

Fig. 1. A sketch of an ultra-small grain coupled to a lead and to a quantum box.

$$H = \sum_{\alpha=L,B}\sum_{k,\sigma}\epsilon_{\alpha k}c^\dagger_{\alpha k\sigma}c_{\alpha k\sigma} + E_C\left(\hat{n}_B - N_B\right)^2 \qquad (1)$$
$$+\,\epsilon_d\sum_\sigma d^\dagger_\sigma d_\sigma + U\hat{n}_{d\uparrow}\hat{n}_{d\downarrow} + \sum_{\alpha,k,\sigma} t_\alpha\left\{c^\dagger_{\alpha k\sigma}d_\sigma + \text{H.c.}\right\},$$

where $c^\dagger_{Lk\sigma}$ ($c^\dagger_{Bk\sigma}$) creates a lead (box) electron with momentum k and spin projection σ, t_L (t_B) is the tun-

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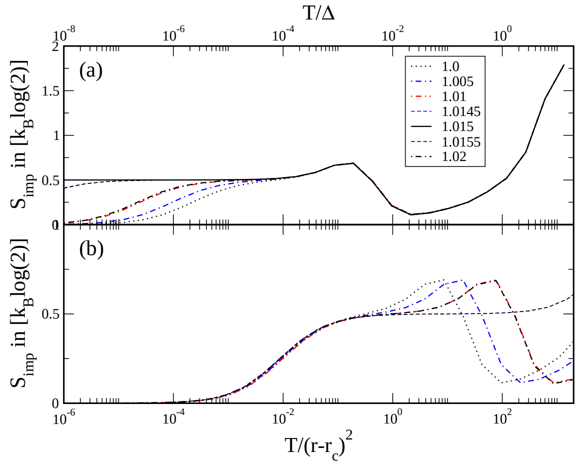

Fig. 2. (a) The dot entropy $S_{\rm imp} = S_{full} - S_{free}$ vs temperature, for different values of $r = t_B/t_L$. (b) $S_{\rm imp}$ replotted vs the rescaled temperature $T/(r - r_c)^2$. Parameters: $D = 10$, $N_B = 0$, $\epsilon_d = -1$, $U = 2$, $E_C = 0.01$, $N_s = 1200$, and $\Lambda = 5$.

neling matrix element between the quantum dot and the lead (box), and ϵ_{Lk} (ϵ_{Bk}) are the single-particle levels in the lead (box). The excess number of electrons inside the box, $\hat{n}_B$, is controlled by the dimensionless gate voltage, N_B. Taking the single-particle levels in the lead and box to have a common rectangular density of states, $\rho(\epsilon) = \rho\theta(D - |\epsilon|)$, we define the energy scale $\Delta = \pi\rho t_L^2$ and use it as our unit of energy.

The Hamiltonian of Eq. (1) is solved using a recent adaptation of Wilson's NRG [7] to the Coulomb blockade [8]. The main complication with applying the NRG to this system stems from the long-range interactions in energy space induced by the charging energy E_C. We circumvent this problem by mapping the original model onto an equivalent Hamiltonian, describing two noninteracting bands coupled to a complex impurity [8]. The resulting quantum impurity problem is then accurately solved using the NRG.

Results: The impurity contribution to the entropy, $S_{\rm imp}$, is defined as the difference between the entropy of (1) with and without the ultrasmall dot present. Figure 2 (a) shows $S_{\rm imp}(T)$ for different ratios $r = t_B/t_L$ near the quantum critical point (QCP) $r_c = 1.015$. The dot parameters were chosen such that the crossover temperature to the single-channel strong-coupling fixed point, T_s, is larger than the charging energy E_C, while the box is tuned to the plateau regime, $N_B = 0$. With decreasing temperature, $S_{\rm imp}$ decreases rather rapidly from the free-orbital value of $S_{\rm imp} = \log(4)$ towards the strong-coupling fixed-point value of $S_{\rm imp} \to 0$. However, since the strong-coupling fixed point is unstable with respect to E_C, the entropy $S_{\rm imp}$ increases again as T crosses E_C [6]. The relevant operators associated with E_C have the effect of both inducing an effective local moment, and suppressing

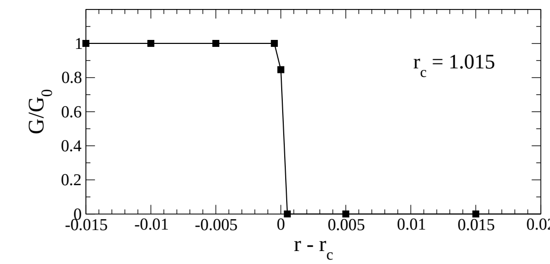

Fig. 3. $T = 0$ conductance as function of $r = t_B/t_L$, for the same parameters as in Fig. 2. Here G_0 is optimal conductance.

the scattering between the lead and the quantum box. At the QCP, the resulting Kondo couplings between the effective moment and the two electron gases are of equal strength. Hence the Hamiltonian flows to the two-channel fixed point, characterized by a residual entropy of $\frac{1}{2}\log(2)$. The two-channel fixed point is unstable against a channel-symmetry-breaking field, induced by a ratio r that deviates from r_c. Plot (b) in Fig. 2 shows the scaling behavior of $S_{\rm imp}$ as a function of the scaling variable $T/(r - r_c)^2$, using the data of (a). We report perfect scaling for $T \ll E_C$, while the crossover regime at intermediate and high temperatures is governed by E_C and T_s. Note that the overall behavior of $S_{\rm imp}$ resembles the large-N limit of Ref. [6].

The different nature of the two Fermi liquids on either side of r_c is illustrated by the conductance in Fig. 3. Now we drive a current through the ultrasmall dot using two noninteracting leads [5]. For $r < r_c$, the effective coupling to the quantum box scales to zero, rendering the conductance perfect at $T = 0$. For $r > r_c$, the coupling to the leads scales to zero and the conductance vanishes. Qualitatively, we find the same behavior when the gate voltage N_B is varied; details will be published elsewhere.

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